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ORIGINAL ARTICLE

Fertility and development: the roles of schooling and family production

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Abstract This paper presents a quantitative theory of development that highlights three mechanisms that relate schooling, fertility, and growth. First, we point out that in the early stages of development, fertility and schooling may rise together as the schooling of younger children increases their relative contribution to family income when they turn working age. Second, the model contains a supply-side theory of schooling that generates a rise in schooling independent of technological change. Third, we introduce a direct negative effect of industrialization on fertility that does not operate through human capital and the quantity-quality tradeoff. An initial quantitative assessment of the theoretical mechanisms is conducted by calibrating and applying the model to United States history from 1800 to 2000. We find that the demise in family production is an important factor reducing fertility in the 19th century and schooling of older children is dominant factor reducing fertility in the 20th century. The same model is applied to England from 1740 to 1940, where we offer two complimentary explanations for the rise in fertility from 1740 to 1820. The first is based on the rapid expansion in the cottage industry and the second on the increased relative productivity of children. We also find that the subsequent fall in fertility from 1820 to 1940 cannot be explained without introducing child labor/compulsory schooling laws.

Keywords Economic development · Fertility · Schooling · Family production

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1 Introduction

Modern economic growth has been universally associated with industrialization, a fall in fertility, and a rise in schooling. This paper re-examines the reasons for these broad features of economic development and offers some new explanations for their connections. In particular, we focus on two aspects of development that have not received sufficient attention.

First, a rise in the level of early primary schooling and basic literacy is generally *not* associated with a decline in a country's average rate of fertility. Instead, it is the rise in late-primary and secondary schooling that is *negatively* correlated with fertility. We offer an explanation that distinguishes between the different effects on fertility of schooling younger children and schooling older children.[1] Greater schooling for older children in the previous generation raises the productivity of parents in the current generation relative to the productivity of their working-age children. This causes older-children's work to become a relatively less important source of family income. The decline in the relative importance of child labor raises the net cost of children, reduces the opportunity cost of education and raises the return to schooling, causing fertility to decline and schooling of older children to rise further over time.[2] In contrast, a rise in the schooling of younger children can actually cause a *rise* in fertility, as this raises the earnings potential of these children when they are older and able to work to support family consumption.[3]

Second, there is a relatively strong and coincident negative association between the industrialization of a country and its demographic transition. The reasons for this association offered in the literature are based on the reasonable premise that industrialization increases the return to human capital and encourages parents to choose quality over quantity.[4] We complement this mechanism with a direct connection between industrialization and fertility that does not work through human capital

1 Galor and Weil (2000) explain the positive correlation between schooling and fertility in the early stages of development by introducing a feature that offsets the quantity-quality tradeoff between schooling and fertility. In their Post-Malthusian Regime there is a period where education begins to rise, and for a given level of household income, this causes fertility to fall (the quantity-quality tradeoff). However, as long as a subsistence constraint on consumption still binds, all marginal changes in household income are spent on children. This "income effect" dominates the quantity-quality tradeoff, and allows fertility and schooling to increase together.

2 This is a "supply-side" mechanism for the expansion in schooling that complements the "demand-side" mechanisms, mentioned below in footnote 4, that raise the return to schooling.

3 Gary Becker, the widely recognized father of the quantity-quality tradeoff, actually preferred the term quantity-quality *interaction* (Becker, 1960, 1981; Becker & Lewis, 1973, 1981). He explicitly acknowledged that the interaction could be positive. "The net cost of children is reduced if they contribute to family income by performing household chores, working in family business, or working in the market place. Then an increase in "earning' potential of children would increase the demand for children (Becker, 1981, p.96)." Increased schooling of young children, who do not yet have the capacity to work, clearly increases their earning potential when they are older and able to generate income for the family. This lowers the net cost of children and increases fertility.

4 Galor and Weil (2000) use a one sector model where the return to human capital is increasing in technological change. Doepke (2004) models industrialization explicitly in a two-sector model with a traditional and modern sector, as in Hansen and Prescott (2002). However, the presence of two sectors only affects fertility indirectly by introducing a second feature that drives up the return to human capital. He assumes that human capital is only productive in the modern sector, so as the economy industrializes the return to human capital rises. These mechanisms are consistent with the ones in this paper, as they all stress the importance of technological change in raising the return to children and reducing fertility.

and the quantity-quality trade-off. Our direct connection helps explain reductions in fertility during periods of industrialization when the schooling of older children is not rising significantly (e.g. the US over the 19th century).

In our model the early stages of development are characterized by “family production”—that is a small business where the owner/manager is an essential and limiting factor in the production technology. Older households possess a family production technology that is learned and passed down when younger members of the family work for their parents. The non-wage income from operating the family business creates a source of wealth that raises the demand for children and lowers the need to save for retirement, both of which slow economic growth.

A movement away from family production, “industrialization,” is needed to increase growth. Adult children and grandchildren also have the option of supplying labor to a “firm,” i.e. a production technology where a particular owner/manger is not a limiting factor—managers can be hired as part of the overall work force. Forces that drive up the market wage, beyond productivity gains in the family production sector, will lead to the demise of family production. Such forces will generate a mechanism for growth by lowering fertility, raising saving, and shifting labor to the faster growing formal or firm-sector.[5]

To get an initial assessment of the quantitative importance of these two mechanisms, we apply the model to United States history. The model is calibrated to have certain steady state properties and to match certain facts about fertility, schooling, interest rates, and employment at the beginning of the nineteenth century. Then a transition path for fertility, schooling, interest rates, and labor productivity growth is simulated for the period 1800–2000. We find the model is able to closely match the general trends in three variables: (i) a downward trend in fertility throughout, (ii) little change in schooling until the 20th century and then a sharp increase, and (iii) gradual increases in growth rates in the nineteenth century, an acceleration in growth rates until the middle of the twentieth century and then a leveling off. We find that the demise in family production is an important factor in explaining the 19th-century fertility decline, while the rise in the schooling of older children is the dominant factor in explaining the fertility decline during the 20th century.

Next, we apply the same model to England over the period 1740–1940. This is a more challenging experiment since in England fertility increased before decreasing and because an early schooling period, where only young children are educated, must be included (requiring a regime switch in the modeling). We provide historical and quantitative evidence that the increase in fertility from 1740 to 1820 can be explained by a combination of the expansion in the cottage industry and a rise in the relative productivity of children. However, the model fails to explain the fertility decline after 1840 without introducing child labor laws and compulsory schooling, as in Doepke (2004).

Section 2 further details how the growth mechanisms emphasized here compare to other mechanisms in the literature. Section 3 establishes the two facts that motivate the analysis. Section 4 presents the model. Section 5 calibrates the model to the United

[5] Caselli and Gennailoi (2005) emphasize another negative effect of family production on economic growth. Ownership and management of family production is not determined by merit and thus may lead to inefficient production. Analyzing nepotism would take us too far afield. It requires modeling heterogeneity in managerial skills and incomplete transmission of skills across generations, as well as imperfect credit and contracting. Our approach misses any negative effects on TFP and thus underestimates the negative effects of family production on growth.

States and simulates historical transition paths from 1800 to 2000. Section 6 applies the model to England's experience from 1740 to 1940. Section 7 concludes the paper.

2 Related literature

The concept of *family* production is related to two other concepts in the literature: (i) *home* production (e.g. Benhabib, Rogerson, & Wright, 1991; Gollin, Parente & Rogerson, 2001; Parente, Rogerson, & Wright, 2000) and (ii) *informal* production (e.g. Hansen & Prescott, 2002; Restuccia, 2004). The difference between family production and home production is largely one of emphasis. First, home production models are used to explain the allocation of labor across market (measured) and home (unmeasured) production sectors. In contrast, we assume that goods produced by the family business or farm can be used at home or sold in markets. Second, the home production literature has focused on adult labor inputs within a household containing a single generation. Our focus is on a multigenerational household, where young adults and children supply labor to production technologies that are owned and managed by the old.

In the literature, informal production has been modeled as agricultural production, with land serving as an important fixed input that generates diminishing returns to the other factors of production. We distinguish family production from informal production by focusing on another fixed input that generates diminishing returns, the single owner/manager that operates the technology. We think of this as the key distinction between a small or family business and a traditional neoclassical firm—the owner/manger is crucial to the operation of the family business. While we could label such a technology a "small" business, we think there is an important family connection in how we apply the concept. The fact that younger generations often work for the family business that their parents own allows them to "inherit" the knowledge of how to operate or "run" a small business themselves. The following quote from Ruggles (2005, p. 11) regarding the United States in the 19th century attests to historical relevance of this mechanism, even outside of agriculture.

> "In the nineteenth century, the bulk of men in the high-status occupations were proprietors of one sort or another. Many of these people inherited their businesses from their fathers. To a lesser extent, that was true in mid-status jobs as well; among the common titles in that category were bakers, brickmasons, cabinet-makers, carpenters, and shoemakers, who typically had their own shops in that period. Many craftsmen inherited their occupations from their fathers, and a son who lived with his parents was no doubt more likely to inherit. Sales clerks had especially high rates of co-residence in the nineteenth century; many of them probably worked in their fathers' stores with the expectation of eventual inheritance."

In Sect. 6 we show that family production, thought of in the way just described, affects fertility directly, while the literature's current interpretation of informal production does not.[6]

[6] Broadening the notion of family inheritance, beyond physical assets such as land, to include inheriting the *skills* to operate a family business, serves to broaden the potential applications of the model. For example, Hong Kong and Singapore went through coincident economic transformations and demographic transitions, despite never having economies that were based on agriculture to any significant

There are now many papers that combine fertility and human capital accumulation in models of economic growth (e.g. de la Croix & Doepke,, 2003, Doepke,, 2005, Moav,, 2005, Galor & Moav,, 2002, Galor & Weil,, 1996, 2000, Greenwood & Seshadri,, 2002, 2003, Hazan & Berdugo,, 2002, Lagerloff,, 2006, Tamura,, 2002). None of the papers differentiate the fertility effects from schooling younger versus older children or model the role of an inherited family production technology.

Galor and Weil (1996, 2000), Galor and Moav (2002), and de la Croix and Doepke (2003) do not directly focus on the economic transformation from informal to formal production. Tamura (2002) focuses on the entire span of human history and includes the switch from informal to formal production. However, he abstracts from the details of the economic transformation during the century or more in which both types of production are used. Greenwood and Seshadri (2002, 2003) and Doepke (2004) both use a two-sector approach, where both sectors simultaneously operate, so that the process of industrialization and its effects on fertility can be examined. In these papers, industrialization raises the return to human capital because human capital is only productive in the modern or formal sector. Rising returns to human capital then affect fertility via a quantity-quality tradeoff. We assume that human capital is productive in the family sector as well as the formal sector, so industrialization does not raise the return to human capital. Instead, industrialization has a direct effect on fertility. The decline in residual income from family production lowers wealth and the demand for children, as smaller non-wage income later in life means that more of the household's early income must be saved rather than spent on a larger family.

With regards to theoretical modeling of the quantity-quality trade-off between human capital and fertility, our model is most similar to Hazan and Berdugo (2002) and Moav, (2005). However, they do not focus on the difference between the schooling of young and old children, there is no family production in their models, and they do not quantify the theory in order to test it against historical data.

Several authors have emphasized the role of child mortality in creating a precautionary demand for children (e.g. Jones,, 2001, Kalemli-Ozcan,, 2002, 2003, Soares,, 2005, Tamura,, 2006), a mechanism not modeled here. Galor (2005) argues that a decline in infant mortality rates could not have been the trigger for the decline in fertility. He points out that the mortality decline started nearly *a century prior* to the fertility decline in Western Europe, and started significantly *after* the fertility decline in the United States. Doepke (2005) provides further criticisms in a detailed examination of the child-mortality approach to explaining the fertility decline.

3 Historical development facts

In this section evidence is provided to support the two stylized development facts highlighted in the introduction: (1) the advance of early schooling and basic literacy *is not* associated with declines in fertility and (2) the economic transformation, away from family production and toward firm production, *is* associated with declines in

Footnote 6 continued
degree. Despite the absence of an agricultural sector, there was nevertheless an economic transformation from informal home-based production to formal firm-based production. In Hong Kong, as late as 1971, 69% of all manufacturing establishments were located in domestic premises. By 1978, that share fell to 44% (Young, 1992, p. 19).

Table 1 Demographic transitions, economic transformations, and schooling

Country	Onset of fertility decline	Onset of economic transformation	Primary enrollment rate at fertility decline %	Literacy rate at fertility decline %	Secondary enrollment rate at fertility decline %
USA	1800–1820	1800–1820	55	60–70	—
England	1820–1830	1800	56	55–60	—
Germany	1870s	1850	64	80	1
Sweden	1880	1850	69	90	1
Japan	1920–1925	1870	65	—	3
Mexico	1960	1950	80	—	11
India	1965	1950	74	—	36
Third world (38-LDCs)	1955–1972	1950	102	—	33

Notes: School enrollment rates and literacy rates are at the date of the onset of the fertility decline. The enrollment rates for the Third World are enrollment rates at the various years the countries began their demographic transitions averaged across the 18 countries. The primary school enrollment rate for Third World countries exceeds 100% because the denominator includes only the population in age-groups that normally attend the primary grades, while the numerator includes actual attendance, including older children that attend primary school. *Sources*: See Appendix

fertility. We also present some historical background for the application of the model to the United States from 1800 to 2000.

3.1 The absence of a negative association between early schooling and fertility

Basic literacy advanced in most countries before there were any signs of an economic transformation or a demographic transition. Educating the population to achieve basic literacy was motivated by several goals—social and national cohesion, enlightenment, religious, and economic reasons alike. Cultural and political differences in the pursuit of these goals caused the timing and extent of public education of young children to vary somewhat across countries (Galor, 2005, pp. 28–34; Cipolla, 1969, pp. 18–22), but the common feature was the desire to obtain basic literacy for much of the population.

The historical evidence that the rise in early schooling and literacy did *not* generate a fall in fertility is given in Table 1. The value in the first column for each country presents the estimated dates at which the demographic transition began. The third and fourth columns give the primary school enrollment rates and the literacy rates at the beginning of the demographic transition.

In all cases, the enrollment rates and/or literacy rates were very high at the onset of the fertility decline. For the early historical periods, the enrollment rates are based on the enrollment rates for those in the 5–14 age-group. Enrollment rates for the 6–9 age-group, those children who dominate early primary school enrollments, would be even higher. This suggests that, prior to the demographic transition, significant progress in early primary education and literacy was achieved without any associated decline in fertility.[7] Crafts (1997) reaches a similar conclusion for the European fertility transition, as do Bongaarts and Watkins (1996) for the fertility transition in today's

[7] It is important to note that the rise in basic learning took place over an *extended* period prior to the onset of the fertility decline (e.g. Clark, 2005, Figure 2; Galor, 2005, Section 2.3.3). There was not a burst in early primary schooling just before the fertility decline.

developing economies. Secondary education, on the other hand, was just beginning to rise at the onset of the fertility decline. Thus, the quantity-quality tradeoff in history is actually a tradeoff between fertility and late-primary and secondary education.

3.2 The close proximity of industrialization and fertility decline

The second column of Table 1 gives the estimated onset of industrialization, when resources shifted away from family production and toward industry, for each country. The economic transformation closely coincides with the fertility decline in all cases. The often quoted exception to the close association between the economic transformation and the demographic transition is England.[8] This is because some historians place the economic transformation earlier (around 1760) and the demographic transition later (around 1870) than we have indicated in Table 1.

However, Crafts (1995, Table 2) provides evidence that TFP did not take-off in England until after 1800. In addition, while it is true that the share of employment in the agricultural sector fell during the 18th century, it was largely offset by increases in family production outside of agriculture. As noted by Sokoloff and Dollar (1997, p.289),

"Cottage manufacture (or putting-out), where workers labored at home as individuals or in family groups for piece rates, was common in England into the late nineteenth century. It was rare in the United States, however, where the overwhelming share of manufactures intended for sale came instead from centralized plants, which operated as manufactories or so-called nonmechanized factories. This mode of manufacturing organization, where workers routinely left home each day to labor together in a structure intended for that purpose, was also used in England but appears to have been much less prevalent".[9]

As for fertility, Woods (2000, p. 72) writes that in England there were "perhaps two demographic revolutions." The first was the rise in fertility from 1750 to the 1820s, when the mean age of marriage among women in England and Wales fell about 3 years. Several historians have recently attributed this rise to the growth of the cottage industry during the first stages of the Industrial Revolution (e.g. Hudson,, 2004, pp. 34–36; Levine, 1977; Schofield,, 2000, p. 57). The second (or conventional) demographic revolution is associated with the significant decline in fertility which began in the 19th century. In terms of the total fertility rate (TFR), the long-term decline in fertility begins after 1820 (Woods, 2000, Figure 1.1; Clark, 2005, Figure 1). We interpret this decline in fertility after 1820 to be the result of an acceleration in TFP, which gave rise to more industrial production (Galor, Figure 2.14) and less family production, and to the rise in the schooling of older children. It should be noted

[8] France is actually a clear exception, but not in a way that contradicts the role of the economic transformation. In France, the demographic transition came well *before* the economic transformation (about the time of the French Revolution—more than 50 years before the economic transformation). There is no generally accepted explanation for the early onset of the demographic transition in France (e.g. Armengaud,, 1976, p. 57; Knodel & Van de Walle, 1986, p. 396; Sharlin, 1986, pp. 234–235). One theory attributes the decline to laws that limited the use of wet-nurses (Hill, Johnston, Campbell, & Birdsell, 1987). Our theory sheds no new light here.

[9] Sokoloff and Dollar argue that the greater importance of cottage industries in England was driven by the higher ratio of labor to land there. This resulted in a much greater share of English agriculture in grains, which have more extreme seasonal demands for labor than did the more diversified American agriculture. For this reason, there was greater scope for and participation in off-season production in England.

that some demographers, including Woods, prefer to date the "second" demographic transition from after 1860, when fertility control within marriage becomes more important than changes in nuptuality for determining fertility. However, in the spirit of Malthus' preventive check, we view changes in nuptuality as also reflecting conscious fertility control and date the transition from the measured decline in total fertility. That the economic considerations we emphasize influenced fertility by affecting the timing of marriage is supported by the following quote from Schofield (1985).

"Accordingly, arguments from changes in real wages, or the structure of employment, to variations in marriage behaviour need to take into account not only earnings available to both partners in years prior to marriage, but also throughout their adult life, including contributions to the family budget that could be expected to accrue from children. It has been argued that such a structural change occurred during the 18th century, as a greater fraction of the population found employment in rural domestic handicraft production and agriculture, thereby causing the overall age at marriage to fall. (p.17–18)."

3.3 Historical background for the United States

In this section we set the historical background for the United States during the period 1800–2000 in more detail. In particular we want to establish the importance of family production in the 19th century and to show that the modest rise in schooling over this century was primarily to spread basic literacy.

3.3.1 Importance of family production

The family, and not the firm, was the predominant center for production in the United States during the nineteenth century (Carter, Ransom, & Sutch, 2003; Ruggles,, 2001). Although the amount of hard data is limited, economic historians argue that family production was ubiquitous prior to and in the early stages of industrialization (Atack, Bateman, & Parker, 2000 a, p. 263; Lamoreaux, 2003; Margo, 2000, p. 232).[10]

One way to gage the prevalence of informal or family production is to examine the fraction of the workforce that received formal wages and salaries. The earliest published census estimates of the proportion of wage and salary workers is for 1850, when this figure was a still-modest 45% for men (Ruggles 2001). This figure was undoubtedly *much* lower earlier in the century. As an initial suggestion, note that from 1850 this figure increases by 5% of all employed men for each of the next several decades, before slowing. Extrapolating backwards, this 5% per decade rate of increase suggests a figure of no more than 20% in 1800.

[10] Slavery, which was legal in the South, has ambiguous implications for the importance of family production. The use of slaves would raise TFP in informal production, but it would raise TFP in more traditional firms that use slaves as well. It is also possible that slavery may have lessened the reliance on family labor and reduced fertility. However, our theory does not argue that fertility was primarily the result of labor constraints. Instead we argue that the greater the portion of family wealth that is derived from business ownership, rather than wage earnings (which increases the cost of raising children), the greater is the demand for children. Regardless of what the connection between family production and slavery may be, there is evidence that family production exerts a force on fertility *independent of slavery*. Carter et al., (2003) run cross-sectional fertility regressions for the United States in 1840. They use two variables that are inversely related to family production: the fraction of the state population in urban areas and the fraction of employment in non-agricultural jobs. They find the effect of both these variables on fertility is negative and highly significant, even in regressions that included a Southern-state dummy and the fraction of the state population that were slaves.

It is also clear that family production was by no means limited to agriculture. Even for mid-century Ruggles provides this characterization of the relationship between manufacturing and family production: "Despite the early growth of the factory system, even manufacturing was still mostly carried out within the household: artisans and their families typically lived together adjacent to the shop where they produced such products as leather goods, flour or furniture. The system of household production also predominated in the service sector, especially in retail trade (2001, p. 12)."

Powerful forces led to the decline of family production. The transportation revolution in the early Republic expanded markets while the dissemination of industrial knowledge from England encouraged production within firms.[11] Families were increasingly drawn into the market economy, selling ever-larger shares of their produce in exchange for market goods. Further, as productivity grew more rapidly in the firm than family sector, relative wages rose off the farm, making it more costly to purchase old-age support from sons.

3.3.2 Nineteenth century human capital growth

In this section we want to establish that there wasn't much increase in the schooling of older children over the entire 19th century. There must have been another factor driving fertility down over this period. We argue that it was the decline in family production.

As evident from Table 1, literacy was quite high in the United States even as early as 1800. There was a rise in formal schooling in the nineteenth century, perhaps even before the movement to free public schools in the 1840s. For example, enrollment in *public* schools in the state of New York rose from 47.5% of all children from ages 0 to 19 in 1815 to 59.9% in 1850 (Randal, 1871). However, at least in large cities, the rise of public school enrollments appears to have been largely a substitution away from *private* enrollments, with little change in the total. In particular, for New York City in 1796, 24.7% of children 0–19 were enrolled, with 89.7% of all enrollments in private schools. In 1850, the *total* enrollment rate had risen modestly to 26.3%, even as private enrollments plummeted to only 18.3% of the total (Kaestle & Vinovskis, 1980, Table 2.3).

Between 1840 and 1880, various measures reveal a slight increase in education (cf. KV, p. 39). This increase appears to have been greatest among children between the ages of 8–13 when literacy skills might be consolidated. Indeed, Goldin and Katz (2003) report that white children above age thirteen were *less* likely to be enrolled in school in 1880 than 1850. In the Northeast age-16 enrollment rates were roughly 38% in 1880, though they had been about 45% in 1850 (their Figure 4.3). Further, over this same period there was a significant increase in the proportion of females in total white enrollments nationally among students age 13 and older. Clearly, a smaller proportion of white teenage males were enrolled in 1880 than in 1850. Of those enrolled, however, days attended per year showed some increase (KV, 39).

Overall, the historical account suggests there was a spread of basic literacy among women and poorer families. However, the evidence also shows little increase in the

[11] Figure 9.1 in Walton and Rockoff (2002, p. 181), shows a roughly 95% decline in upstream river rates per ton mile between 1790 and 1824, a 90% decline in canal rates by 1850, and a roughly 60% decline in railroad rates between 1830 and 1864. Simultaneously, the speed of transport also increased dramatically.

Table 2 Fraction of year spent in school (0–19 year-olds)

1850*	0.08
1860*	0.09
1870	0.09
1880	0.10
1900	0.11
1920	0.16
1940	0.25
1960	0.29
1980	0.29
1994**	0.31

Notes: Based on data from Goldin, (1999). *Assumes days attended per enrolled student are equal to 1870. **Assumes days attended per enrolled student are equal to 1980

education of most white males over the entire century. This implies that the motivation for education did not exceed basic literacy.

To quantify the extent of schooling, we estimate the time spent in school by 0–19 year-old children using data on white enrollment rates for 5–19 year olds and data on days attending school per enrolled student, both found in Goldin, (1999). We calculate the average fraction of the year attending school for 5–19 year-olds and then multiply by 0.75 to account for the fact that 0–4 year olds did not attend school. The results are presented in Table 2.

Time spent in school changed very little from 1850 to 1900. While the data does not go back to 1800, we suspect it did rise somewhat over the period 1800 to 1850. However, as the historical analysis indicated, much of this rise was merely a substitute for informal learning that generated the relatively high literacy rates at the beginning of the century. The low values for time spent in school by children under 19 in 1900 also supports the notion that very few older children attended school in the 19th century.

4 The model

In this section we present a simple overlapping-generations model, extended to include schooling, fertility, and family production.

All households live for three periods; one period of childhood and two periods of adulthood. Households value their consumption over the two periods of adulthood (c_t^y, c_{t+1}^o) and the adult earnings of all their n_{t+1} children. Adult earnings are determined by the market rental rate on human capital times the adult's human capital stock $(w_{t+1}h_{t+1})$. Preferences are given by $\ln c_t^y + \beta \ln c_{t+1}^o + \psi \ln \left(n_{t+1}w_{t+1}h_{t+1}\right)$ where β and ψ are preference parameters.

Adults inelastically supply one unit of labor when young and zero units when old. Young adults decide how many children to have in the first period of adulthood. Children have an endowment of $T < 1$ units of time that they can use to attend school or work. Children have less than one unit of time to spend productively because in the very beginning years of childhood they are too young to either attend school or to work, and in the middle years they do not have the mental or physical endurance to school or work as long as an adult.

Final goods are produced in two sectors. There is a standard formal sector, where perfectly competitive firms hire labor at the competitive wage rate, w_t, and rent physical capital at the competitive after-tax rental rate, r_t, to produce goods and services. There is also an informal family sector that produces the same goods and services. An old household owns a family production technology. The labor input hired by an old household (f_{t+1}) comes from the younger generations of workers, which may include the old household's own children and grandchildren, as well as workers from outside the family. Since all workers have an outside option to work in the formal sector, they must receive payment equal to the market wage in the informal sector.[12] Let $\omega_{t+1} \equiv w_{t+1} f_{t+1}$ denote the wage bill of the family producer.

The family production technology is

$$O^f_{t+1} = A_{t+1} f_{t+1}^{1-\rho}, \tag{1}$$

where A_t is the disembodied level of technology associated with family production and $0 < \rho < 1$ is a technology parameter. The young workers "learn" the family production technology from their experience as workers. They thus "inherit" the family technology from their parents and other older adults who might hire them.[13]

While children may work as they become older, they are also expensive to care for and feed. To raise each child requires a loss of adult consumption equal to a fixed fraction τ of the adult's first period wages.[14] We assume that the costs of having a child exceed the income generated for the household by child labor. Thus, there is always a net cost to having another child.[15]

Parents determine their children's schooling. Childhood is divided into two subperiods: early childhood, when the child is unable to work, and later childhood, when the child is of working age. The time spent learning in early childhood is $\bar{s}_t$ and in later childhood is $\tilde{s}_t$. The full time available to educate a young child is $\bar{s}$, so $\bar{s}_t \leqslant \bar{s}$. The total time available for an older child to work and learn is $T - \bar{s}$. The early learning of young children gives older children $\bar{h}_t = \gamma \bar{s}_t^{\theta}$ units of human capital that can be used in production during the later years of childhood, where $0 < \theta < 1$ is a parameter that gauges the effect of schooling on human capital accumulation and $0 < \gamma < 1$ reflects the fact that children lack relative physical strength or experience in applying knowledge to production compared to an adult. Adult human capital of the same person in the next period is $h_{t+1} = \bar{h}_t \left(1 + \Phi \left(\tilde{s}_t + \eta\right)^{\phi}\right)$, where $\Phi > 0$, $\eta > 0$, and $0 < \phi < 1$. We define $1 + \Phi\eta^{\phi} \equiv 1/\gamma > 1$, so that, for reasons stated above, adult human capital exceeds that of a child worker even when years of schooling are the same.

[12] Introducing altruism from children to parents would create a wage gap between working at home and working at firms. Children would accept lower wages from parents and grandparents than from firms. However, the basic mechanism would remain the same—technological change that favors formal production in firms raises the market wage and the opportunity cost of working for the family.

[13] While we do not explicitly model land as a productive input, one can think of land as being part of the family technology. The fact that land must be split across several children would work against technological innovations that increase family productivity and thus slow the growth of A over time. We also considered versions of the model with endogenous family capital. This added no further insight and little improvement in the model's ability to match the data.

[14] We assume that the cost of children directly involves forgone consumption rather than indirectly through forgone earnings. The results are qualitatively identical and quantitatively very similar under the two interpretations. Assuming direct forgone consumption simplifies the notation slightly.

[15] This is consistent with Craig's (1993) finding that farm children had negative net present value in the United States during the mid-19th century.

Schooling is provided by non-profit institutions, such as the church or the government. The direct private cost of schooling paid by households is p_t units of consumption goods per unit of time spent in school. The portion of schooling costs subsidized by the government is financed by a tax on capital income. More specifically a capital tax rate μ_t is levied on both the gross income earned from renting capital to firms and the residual income earned, after labor costs, from operating a family business.[16] When educating older children, in addition to the direct cost of schooling, the household also bears the indirect cost from the forgone earnings of reduced child labor.

4.1 Household decisions

The generation-t household chooses $\bar{s}_t, \tilde{s}_t, n_{t+1}, f_{t+1}$, and a life-cycle consumption path, to maximize lifetime utility subject to the lifetime budget constraint,

$$c_t^y + \frac{c_{t+1}^o}{1+r_{t+1}} + n_{t+1}(\tau w_t h_t + p_t(\bar{s} + \tilde{s}_t)) = w_t h_t + n_{t+1} w_t \bar{h}_t \left(T - \bar{s} - \tilde{s}_t\right) + \frac{(1-\mu_{t+1})\left(O_{t+1}^f - \omega_{t+1}\right)}{1+r_{t+1}}.$$

Proposition 1 shows that there are two distinct regimes of household behavior (all proofs are in Appendix A). In the first regime (*Y-regime*), the household only schools its children when they are too young to work, and in the second regime (*O-regime*), younger children spend the maximum amount of time in school and older children spend positive amounts of time in school. The household moves from one regime to the other, when the direct private cost of schooling becomes sufficiently low.

Proposition 1 *Under the following assumptions about parameters values,*

(A1) $\frac{\phi(1-\gamma)}{\eta} = \frac{\theta\tau + \bar{s}\gamma(1-\theta)}{\bar{s}(\tau - \gamma(1-\theta)(T-\bar{s}))}$,
(A2) $\tau - \gamma T > 0$,

there are two distinct schooling regimes. In the Y- regime, $\bar{s}_t < \bar{s}$ *and* $\tilde{s}_t = 0$*, and in O- regime,* $\bar{s}_t = \bar{s}$ *and* $\tilde{s}_t \geqslant 0$*. Furthermore, one moves from the Y-regime to the O- regime, as* $p_t/w_t h_t$ *falls below the critical value,* $\frac{\theta\tau}{\bar{s}(1-\theta)}$.

The first parameter-assumption (A1) guarantees that while $\bar{s}_t < \bar{s}$, and schooling of young children is rising, there is no schooling of older children. When the private price of schooling falls to the point that $p_t/w_t h_t = \frac{\theta\tau}{\bar{s}(1-\theta)}$, then $\bar{s}_t = \bar{s}$, but schooling of older children remains at zero. The second parameter-assumption (A2) guarantees that the schooling of older children will rise once $p_t/w_t h_t$ falls below $\frac{\theta\tau}{\bar{s}(1-\theta)}$. Without (A2), schooling of older children does not necessarily rise with decreasing school prices because a lower price of schooling lowers the cost of children and may increase the number of children. The effect of having more children may decrease schooling per child. Assumption (A2) insures that the fertility effect of falling school prices will not dominate, allowing schooling to rise as the private price of schooling falls.

The next proposition gives the household demand for schooling and children in the two regimes.

16 The capital tax mimics the property tax that was predominately used to finance schooling in the U.S. Under the assumptions that we use when applying the model, the capital tax has the convenient property that no private behavior is affected by it.

Proposition 2 *Household demand functions for children and schooling differ across the two regimes and are given below. However, at $\bar{s}_t = \bar{s}$ the two demand functions for children yield the same number of children, so that fertility experiences no discrete changes as the family moves across regimes.*

Y- Regime—Schooling of younger children only

$$\text{(2a)}\quad n_{t+1} = \frac{(1-\theta)\psi\left(1+\frac{\rho\left(1-\mu_{t+1}\right)O^f_{t+1}}{w_t h_t\left(1+r_{t+1}\right)}\right)}{(1+\beta+\psi)\left(\tau-(1-\theta)\gamma(T-\bar{s})\left(\bar{s}_t/\bar{s}_{t-1}\right)^{\theta}\right)},$$

$$\text{(2b)}\quad \bar{s}_t = \frac{\theta\tau(w_t/p_t)\bar{s}^{\theta}_{t-1}}{(1-\theta)}.$$

O- Regime—Schooling of older children

$$\text{(3a)}\quad n_{t+1} = \frac{\left(1-\frac{\phi}{1+\frac{1}{\Phi\left(\tilde{s}_t+\eta\right)^{\phi}}}\right)\psi\left(1+\frac{\rho\left(1-\mu_{t+1}\right)O^f_{t+1}}{w_t h_t\left(1+r_{t+1}\right)}\right)}{(1+\beta+\psi)\left(\tau+\frac{p_t(\bar{s}-\eta)}{w_t h_t}-\frac{T+\eta-\bar{s}}{1+\Phi\left(\tilde{s}_{t-1}+\eta\right)^{\phi}}\right)},$$

$$\text{(3b)}\quad \tilde{s}_t = g\left(\tilde{s}_{t-1}, \frac{p_t}{w_t h_t}\right),$$

where g is increasing in $\tilde{s}_{t-1}$ and decreasing in $p_t/w_t h_t$.

First, note that in either regime if (i) the relative price of schooling ($p_t/w_t h_t$) and the level of schooling are both constant and (ii) there is no family production, then fertility would be a constant. Thus, the functional forms admit no effect from increases in wages, independent of schooling, on fertility. This is because the wealth and substitution effects of wages on fertility exactly offset.[17]

Second, focus on the effects of schooling by continuing to assume no family production. During the early schooling regime, (2a) indicates that fertility is increasing in the *growth rate* of schooling ($\bar{s}_t/\bar{s}_{t-1}$). A higher growth rate means working-age children are becoming more productive compared to parents to a greater degree over time. This raises children's relative contribution to family income and lowers their net cost, causing higher fertility.

Over early periods of development when $\bar{s}_t$ is rising, one would not see a decline in fertility unless the growth rate of schooling is falling. This will be the tendency, other things constant, because of the neoclassical dynamics present in (2b) (driven by diminishing returns to schooling). However, if $\bar{s}_t$ is rising due to a rise in w_t (increased physical capital intensity or increased technological change) or a fall in p_t, the growth rate of schooling may not be declining until $\bar{s}$ is reached. Thus, over early periods of development, as schooling is rising, fertility may be constant or rising.

In contrast, during the regime where older children are being schooled, schooling and fertility must be negatively correlated (in the absence of family production). As indicated by (3a), fertility falls as the human capital of parents rises relative to the human capital of older children (i.e. as $\Phi\left(\tilde{s}_t+\eta\right)^{\phi}$ increases). Greater schooling of older children by the current generation raises their adult earnings in the next period *relative to their children's potential earnings at working age*. This raises the net cost of having children, so fertility declines. Thus, fertility is driven down by the rise in schooling of older children.

17 This property is a common feature of many theories of fertility. See, for example, Becker and Barro (1988), Barro and Becker (1989), and Alvarez (1999).

Now consider the effects of family production. Family production introduces a new term, $\rho\left(1-\mu_{t+1}\right) O_{t+1} / w_t h_t\,(1+r)$, that raises fertility (other things constant). The numerator of this term is the after-tax share of family production that flows to older households ($\rho O_{t+1} = O_{t+1} - \omega_{t+1}$). The denominator is the potential "full" wage that can be earned as a young adult, which determines the opportunity cost of having children. The more important family production is, relative to the opportunity cost of children, the stronger is the demand for children. This is not a pure wealth effect, but rather is an effect that arises when one form of wealth (that does *not* affect the net cost of children), the ownership of family production, changes relative to another form of wealth (that *does* affect the net cost of children), the flow of adult earnings. A shift in the composition of family wealth away from family production and toward adult labor income causes the net cost of children to rise, for a given level of family wealth, and the demand for children falls.[18]

There is no effect of family production on schooling because of two offsetting effects. To see these effects, first note that fertility raises the cost of schooling children (more children means greater forgone consumption of parents as schooling rises and child labor income falls). Second, note that the level of parental consumption determines the marginal *value* of forgone consumption associated with greater schooling (higher parental consumption levels means parents can better "afford" the lost consumption associated with more schooling). Family production raises *both* fertility and parental consumption, other things constant. As just mentioned, higher fertility lowers the incentive to school children, but a higher consumption level raises the incentive to school children. With our functional forms for preferences and human capital production, these two effects always exactly offset.

Note that greater schooling raises the full wage and lowers the relative value of family production, offering a new dimension to the quality-quantity trade-off. More importantly, there is now a mechanism for lowering fertility that is *independent* of the level of schooling. The relative importance of family production falls if "family-based" technological progress (A) rises more slowly than the "firm-based" technological progress that determines the growth in the market rental rate on human capital (w). So, even if schooling is relatively constant, fertility will decline if technological progress in firms outpaces that in families.

4.2 Competitive equilibrium

As mentioned, production also takes place within standard neoclassical firms that combine physical capital (K_t) and human capital (H_t) to produce output from a

[18] Suppose that instead of family production we introduce informal production as in Hansen and Prescott (2002). Informal production involves introducing a second sector where land is the limiting input. The second sector is also operated by firms, and not families, that rent land and hire labor from households. Furthermore, the land is not inherited or passed on from one generation to the next, but rather is purchased by the young and sold by the old in each period (as is the case for physical capital). Since the rate of return on land must equal the interest rate, if both land and capital are held in equilibrium, the *lifetime* budget constraint of the household is unaffected and would take the same form as the constraint in the absence of family production. In this case, the numerators of (2a) and (3a) would contain no family production terms and fertility would be determined solely by schooling. Thus, the presence of land would not affect fertility. To get (2a) and (3a), there must be a second period source of income from operating a family business with the owner/manager serving as the limiting factor. Of course, if one thinks of inheriting the family business as a direct inheritance of assets, this could include the inheritance of land. However, we have just demonstrated that an explicit land market is not essential to the logic, so we have chosen to abstract from it.

Cobb-Douglas technology

$$O_t = K_t^{\alpha}\left(D_t H_t\right)^{1-\alpha}, \tag{4}$$

where D_t is the disembodied technology associated with production in firms, which grows at the exogenous rate d_t. Firms operate in perfectly competitive factor and output markets. This implies the profit-maximizing factor mix must satisfy

$$r_t + \delta = \left(1-\mu_t\right)\alpha k_t^{\alpha-1}, \tag{5a}$$

$$w_t = D_t\left(1-\alpha\right)k_t^{\alpha}, \tag{5b}$$

where δ is the rate of depreciation on physical capital and $k \equiv K/DH$.

For factor markets to clear, the physical and human capital demanded by firms must equal the savings and human capital supplied by adult workers. In addition, the labor demanded by family producers must equal the labor supply of children plus the adult labor not supplied to firms. The market clearing conditions are

$$K_{t+1} = N_t a_{t+1} \tag{6a}$$

$$H_t = N_t l_t h_t \tag{6b}$$

$$N_{t-1} f_t = N_t h_t\left(1-l_t\right) + N_{t+1}\bar{h}_t\left(T-\bar{s}-\tilde{s}_{t+1}\right)\left(1-m_t\right), \tag{6c}$$

where $a_{t+1} = \left[\frac{\beta}{1+\beta+\psi} - \frac{1+\psi}{1+\beta+\psi}\left(\frac{O^f_{t+1}}{w_t h_t(1+r_{t+1})}\right)\right] w_t h_t$ is the physical capital supplied by each young household, N_t is the size of generation-t, and m_t is the fraction of children's available work time spent in the formal market. Note that saving, which would otherwise be a constant fraction of adult wages, is reduced by family production. The second period income flow from family production reduces the need for physical capital accumulation as a source of retirement consumption. Thus, the presence of family production both raises fertility and lowers saving.

In summary, for a given time path of the private price of schooling and the capital income tax, a competitive equilibrium is given by the household behavior in Proposition 2, the firm behavior in (5), and the market clearing conditions in (6).

5 Applying the model to the United States (1800–2000)

Sections 5 and 6 assess the quantitative properties of the model. We do this by trying to replicate the main features of economic growth over long historical periods in the United States and England. To facilitate empirical application of the model, we begin by making the following auxiliary assumptions.

5.1 Auxiliary assumptions

(A3) The private price of schooling is proportional to adult wages (the cost of teachers): $p_t = \pi_t w_t h_t$. The factor of proportionality is exogenously determined by the extent to which schooling is provided and subsidized by churches and the government and by technological advancements that lower the cost of transportation.

(A4) The share of family production that goes to the family producer as residual income is the same as the capital share in the formal sector: $\rho = \alpha$. Evidence suggests that labor's share of national income does not vary with the state of development, and thus does not vary with the mix of informal and formal production methods.

(A5) The rate of depreciation of physical capital is one: $\delta = 1$. Given the long periods in the model (twenty years), this is a reasonable approximation.

(A6) The steady state of the economy is characterized by a zero private price of schooling, full-time schooling of children, and a constant population: $\pi_t = 0, \tilde{s}_t = T - \bar{s}$, and n = 1.

The auxiliary assumptions allow us to state the dynamic competitive equilibrium as system of difference equations.[19] Given the time paths for π_t, A_t, and D_t, the dynamic competitive equilibrium is represented by a difference equation system in $n_t, \bar{s}_t, \tilde{s}_t, k_t, l_t$, with a regime switch at the critical private price of schooling associated with $\pi_t = \theta\tau/1 - \theta 1/\bar{s}$ (see *Appendix A* for a full description of the system).

In this section we calibrate the model to the United States and then simulate values for schooling, fertility, and the returns to physical capital over two centuries (1800–2000). Given these values, we can also simulate the growth rate of labor productivity over the same period. The objective is to see if the theory is broadly consistent with economic growth over the two centuries. It will also produce an initial estimate of how important schooling and family production are in explaining the fertility decline.

5.2 The direct private cost of schooling

Given the widespread literacy and early public support for education observed in 1800 (Lindert,, 2004, p. 123), we assume the United States was in the *O-regime* where families were beginning to educate their older children (albeit very slowly for almost a century). The common-school and high school movements (Goldin, 1999) suggest that throughout the period 1800 – 1960 there was a steady increase in the availability of, and public support for, education.

We interpret the early and steadily increasing support for public education as implying a relatively low private price of schooling in 1800, followed by a steady decrease over the subsequent century and a half. The initial value of π is calibrated to match the initial level of fertility in 1800 (other things constant, fertility varies inversely with π since it raises the cost of children). Women had about 7 children in 1800 (Greenwood & Seshadri,, 2002), so we targeted n, the number of children per adult, to be 3.5, in our first period. The calibrated value for π implies that the fraction of young parents' wages spent on schooling expenses, $p_t s_t / w_t h_t$, was about one percent in 1800. We then assume a linear decline in π_t from 1800 to 1960, where the private cost reaches zero.[20]

[19] We assume that children only supply labor to family production ($m_t = 0$) when we apply the model to the US. We also examined the assumption that children and adults supply the same fraction of their time to the market ($m_t = l_t$) with little change in results. In the subsequent application to England, we also allow for the possibility that child labor can be supplied to firms.

[20] We experimented with alternative price paths and found that one cannot deviate too far from our chosen path. First note that Ruggles's data, and our assumption that $\alpha = \rho$, fixes the influence of family production on fertility. So, alternative fertility patterns are determined by the paths of schooling and schooling prices. If prices fall significantly *faster*, then fertility will fall slower, and may rise, while schooling will rise faster. One can prevent the more rapid rise in schooling, by increasing ϕ (which determines the speed at which returns diminish, and thus the fundamental dynamics of schooling for a given price path). However, preventing a more rapid rise in schooling will make it more likely that fertility will actually rise in the 19th century simulations. On the other hand, if prices fall significantly *slower* then fertility will fall faster and schooling will rise more slowly. To increase the rise in schooling during the 19th century, ϕ will have to be reduced. However, this will cause schooling to rise too fast in the 20th century. In this sense, one can interpret the price path assumption as part of the calibration that helps the model match the schooling and fertility transition paths.

5.3 The work and productivity of children

As mentioned, we think of each period as lasting 20 years. Young adults aged 20–39 are endowed with one unit of time for work. Children aged 0–19 are endowed with one half that amount, i.e. $T = 0.5$. One way to interpret this assumption is that children aged 0–5 can't work at all, those aged 6–14 are only able to average half the hours of an adult, and those aged 15–19 can work as much as an adult.

Even with the same schooling as adults, children are not as productive in a given hour of work because they have less physical strength and experience. In the early 19th century estimates of the relative wage rate (γ) of children less than 16 years old range from 0.20 to 0.37 (Goldin & Sokoloff, 1984, Lebergott, 1964). We set $\gamma = 0.28$. This implies that if a child works the full time endowment T that their contribution to family income is 14% of an adult's. This is close to the estimates of Craig for US 1860 farm families. He found that children below age 6 depress farm output by about the same amount that those ages 6–12 increase it. The contribution of teenage males was just above a quarter that of adult males. Thus, over the entire period of dependency the average annual contribution would be around 10% (a number which excludes any off-farm earnings the child may receive). Similarly, Lindert (1980) estimates that children aged 10–18 contributed an average of 13% to family income in rural England at the end of the 18th century.

5.4 Schooling

Following our historical discussion, we interpret $\bar{s}$ as the average learning time associated with a significant majority of the population obtaining basic literacy. We set $\bar{s}$ to 0.085, about the time investments observed in the US at the beginning of the 19th century (see Table 2). We then interpret schooling over most of the 19th century as spreading basic literacy to the entire population. Literacy rates rose in the U.S. from 60 to 70% in 1800 to over 90% in 1860 (Lindert,, 2004). We interpret an average schooling time of 0.10 – 0.11, as observed in the U.S. between 1860 and 1900, as establishing close to universal literacy.

The parameter ϕ governs how fast schooling rises over time. We set ϕ to generate values for s near 0.10 – 0.11 between 1860 and 1880. We assume that child labor approaches zero over time, so that eventually there is full-time schooling. The parameters τ, η, and Φ are set to satisfy (A1), to generate full-time schooling ($\tilde{s} = T - \bar{s}$) as the unique steady state solution of (8b), and to match the relative productivity of children ($1 + \Phi\eta^{\phi} \equiv 1/\gamma$).[21]

5.5 Technological change

We assumed an annual rate of technological change in firms, the value for d, so as to match the overall rate of TFP growth for the economy in the 20th century. Estimates of the rate of growth of TFP in the 20th century vary because approaches to measuring the growth in physical and human capital vary. We chose a value that kept the average labor productivity growth in the 20th century within the estimated range of

[21] Note that to use (A1) we must also determine a value for θ, even though it is a parameter that is not directly needed to determine the schooling of older children. The value for θ is determined when the model is applied to England in the next section. In this sense, calibrating the complete model involves coordinating features of the US and England.

1.7% (Gordon,, 1999) to 2.2% (Ferguson & Wascher,, 2004). We found that an annual TFP growth rate of 0.5% centered labor productivity growth in this range.

Our method for identifying the relative technological change in family production (A/D) is based on the fraction of the workforce in formally-paid wage and salary employment. We assume that most labor used in family production was informally paid family members. Under this assumption, the rise in formally-paid employment tells us something about the decline in family production. We substitute Ruggles's (2001) estimate of the fraction of workers in paid-employment for each period into (10) and then set the relative TFP in family production, A/D, to satisfy the equation. Ruggles' data on l only goes back as far as 1840. Since l rose at a constant rate for most of the 19th century after 1840, we extrapolated back to get l-values of 0.22 and 0.31 for 1800 and 1820.

5.6 Steady state values

As the adult work force shifts to the formal sector, and as older children shift away from work and toward schooling, the importance of family production approaches zero over time. This implies unique steady state solutions for fertility and physical capital intensity from (8a) and (9). We set the steady state k in the closed economy to produce a return to physical capital of 5%. This produced a value for the return to capital close to 7% toward the end of the 20th century. We set $n = 1$ (zero population growth) in the steady state. These two steady state conditions allow one to calibrate the preference parameters, ψ and β.

5.7 Initial value for k

The initial value of k was set to keep interest rates in 1800 within values that were 2–6% points higher than those simulated for 2000. Wallis (2000, Figure 2) reports that real interest rates on national government debt averaged about 5% in the first half of the 19th century and averaged about 2.5% in the 20th century. Barro, (1993) reports that real interest rates on commercial paper were 9% from 1840 to 1880, but averaged about 3% during the 20th century. The setting of the initial k was also guided by the attempt to keep the rate of return to capital in 1800 close to the endogenous value in 1820. In this sense, we chose an initial value that the model could support without implying a dramatic change after the initial period. The calibrated parameter values are summarized in Table 3.

5.8 Simulations

The calibrated model can now be simulated to produce historical predictions about key variables that were not targeted by the calibration itself. These predictions include the time paths of fertility (n), schooling in the 20th century (s), and labor productivity growth and interest rates for two centuries.

In terms of the actual data, fertility declined over the two centuries, falling to a little under two children per adult in 1900 and to about one child per adult in 2000 (Caplow, Hicks, & Wattenberg, 2001). Labor productivity grew less than one percent annually during the first half of the 19th century and then grew at around one percent until the 20th century (Greenwood & Yorukoglu, 1997). During the 20th century growth rates rose until mid-century and then leveled off (Ferguson & Wascher,, 2004,

Table 3 Calibrated parameter values

Parameter		Target/assumption
γ	0.28	Relative wage of children (1800)
T	0.50	Family share of child's income (1800)
τ	0.14	Steady state schooling
ϕ	0.39	Late 19$^{\text{th}}$ century schooling
$\alpha = \rho$	0.33	Standard value for capital share
η	0.025	(A1)
ψ	0.22	Steady state fertility
β	0.32	Steady state interest rate
Φ	10.83	$1 + \Phi\eta^{\phi} \equiv 1/\gamma$
π_{1800}	0.135	Fertility (1800)

Gordon,, 1999). The average growth rate in labor productivity over the 20th century was between 1.7% and 2.2%. After little change over the 19th century, schooling investments in older children rose significantly over the twentieth century until leveling off in the last quarter century. Table 2 shows the rise in time spent in school was almost three fold over the 20th century; s increased from 0.11 in 1900 to 0.31 in 1994.

Figures 1–4 presents the simulations (dashed-line) against the actual data (solid-line). Schooling grows slowly throughout the 19th century and then begins its upward climb toward the end of the 19th and beginning of the 20th century. The model matches the sharp rise in schooling over the entire 20th century, but at a steeper pace than what was observed late in the century. Fertility declines somewhat slower than we observed over the first century; with a value of 2.4 by 1900 instead of 1.8 as actually observed. This leaves room for omitted factors such as the decline in infant mortality to play a role.

The long-run decline in fertility in the model is from 3.5 to 1 child per adult. If schooling is held constant at its initial value the decline would be from 3.5 to 2.9. Thus, only about 1/4 of the decline (0.6/2.5) is due to the demise of family production. The importance of family production is greater in the 19th century, where it explains about 40% of the decline.

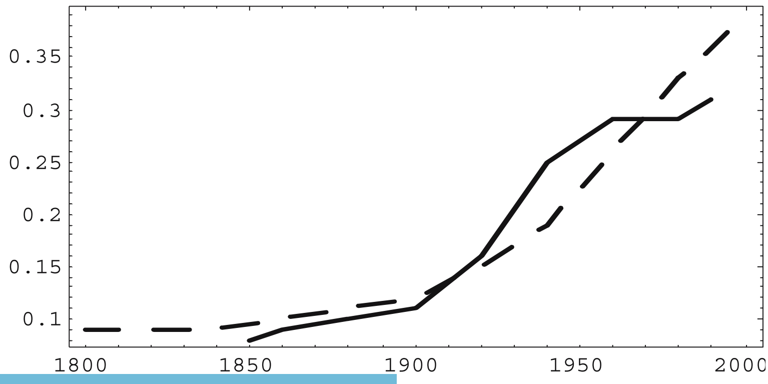

Fig. 1 U.S. schooling 1800–2000. *Notes*: Actual Schooling-solid line, Simulated schooling-dashed line. Actual Schooling from Table 2

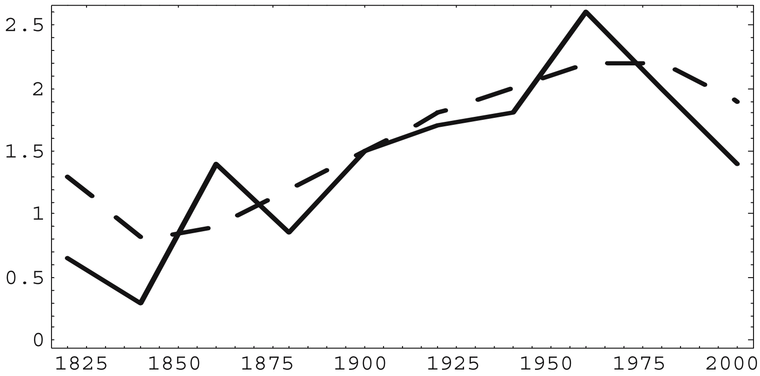

Fig. 2 U.S. labor productivity growth rates 1800–2000. *Notes*: Growth rates in annual percent. Actual growth-solid line, Simulated growth-dashed line. Actual growth rates from Greenwood and Yorukoglu (1974, Figure 5), Gordon (1999, Tables 1 and 2), and Rangazas (2002)

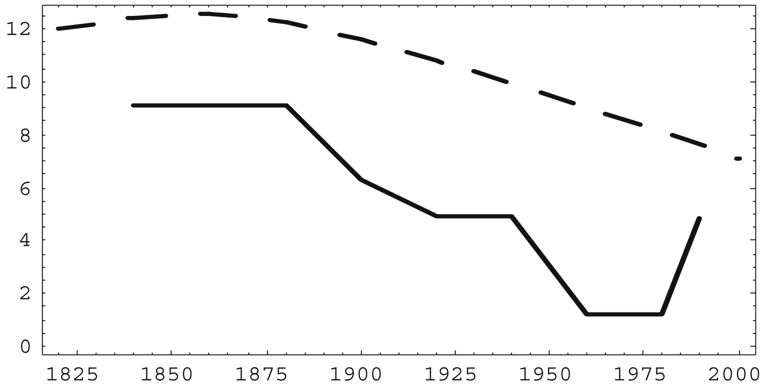

Fig. 3 U.S. interest rates 1800–2000. *Notes*: Interest rates in annual percent. Actual interest rates-solid line, Simulated interest rates-dashed line. Actual interest rate on six-month prime commercial paper from Barro (1993, Table 11.1)

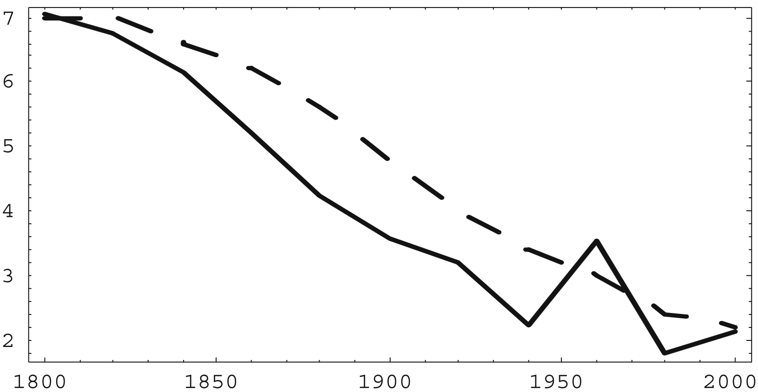

Fig. 4 U.S. fertility 1800–2000. *Notes*: Actual fertility—solid line, simulated fertility—dashed line. Actual fertility from Haines (2000). Fertility is plotted as children per woman as opposed to children per person (the theoretical variable from the model)

The simulated values of labor productivity growth are surprisingly good. Simulated growth rates were near 1% for the 19th century and then trended upward until leveling off from 1940 to 2000. The particular pattern for the growth rates is created by the combination of rising schooling investments and diminishing returns to schooling. Early on, schooling rose rapidly enough that diminishing returns were offset by rising investment and growth rates increased. However, diminishing returns eventually dominated causing growth rates to level off and fall back despite further increases in schooling investments.

The least accurate prediction concerns interest rates. The model predicts a gradual decline in interest rates starting after the Civil War. The starting point of the decline is quite accurate but instead of a gradual decline, actual interest rates fell sharply between 1870 and 1910 and then leveled off. The sharp drop in interest rates was at least in part due to an inflow of foreign capital during the peak of the US industrial revolution. Since we assume a perfectly closed economy, this capital inflow is not captured here.

6 Applying the model to England (1740–1940)

In this section we explore the quantitative properties of the model when there is a transition from the Y-regime to the O-regime. England is a good candidate for this exercise because it is well known that their schooling lagged behind that of the US and other developed countries in the 18th and 19th centuries. In addition, enough data is available for these two centuries to piece together actual trends in key variables.

England's primary schooling and literacy rates were at modest levels in 1740 and rose slowly from 1760 to 1850 (Crafts, 1997, Table 1). While the U.S. achieved almost universal basic literacy among its white population between 1860 and 1880, England did not approach universal adult literacy until the turn of the 20th century (Crafts, 1995, Table 2). Fertility first rose modestly over this period from a TFR of about 4.5 in 1750 to about 5.5 in 1820 (Woods, 2000). Fertility began to fall after 1820, reaching a low of a little over 2 by 1940.

We examine the model's ability to generate these quantitative patterns, where 1740 to 1820 is interpreted as the *Y-regime*, with schooling and fertility both rising, followed by a transition to the *O-regime* in 1840, with a decline in fertility and an increase in the schooling of older children. We consider two reasons why fertility might have risen during the Y-regime. First, the relative productivity in the family sector may have risen due to above average growth in the cottage industry during the first industrial revolution. Second, the relative productivity of children may have risen because agriculture was being replaced by the cottage industry (so that physical strength was less of a constraint on productivity) and employment opportunities opened up outside the home in the early mills. *Experiment #1* examines the effects of a rise in relative TFP in the family sector. *Experiment #2* maintains a constant relative TFP in the family sector and instead examines the effects of a rise in the relative productivity of children (γ).

For each experiment we also need to (i) calibrate the parameter for the early schooling production function, θ, (ii) determine the time path for the relative private price of schooling, π_t, and (iii) set the initial physical capital intensity.

6.1 Parameter calibration

We assume that all the parameters calibrated to United States in the previous section applied to England in this earlier period (with the exception of γ in *Experiment #2* from 1740 to 1820). The parameter θ has direct relevance for the Y-regime and is calibrated to place relative TFP in the family sector (A/D) in the period from 1760 to 1840 at levels that are generally comparable to those that we indirectly measured for the US in 1800. Note from (7a) that θ affects fertility directly, and therefore will indirectly affect the value of A/D needed to match any fertility level. The relative TFP in 1760 was set so that fertility in 1740 matched the observed value of 2.25 children per adult. With $\theta = 0.245$, the value for A/D in 1760 that met the fertility target for 1740 was 0.19, below the US value in 1800 of 0.24. Under assumptions that we defend below, relative TFP in the family sector may have increased from 1760 to 1820, reaching a value of 0.25 in 1820, very close to US values in the early 19th century.

6.2 TFP

In both experiments we set the rate of growth of TFP in the formal sector based on Crafts' (1995, Table 2) estimates. He estimates no growth in TFP from 1740 to 1780, 0.1% growth from 1780 to 1800, 0.4% growth from 1800 to 1820, 0.75% growth from 1830 to 1900, and 0.5% after 1900.

There are no direct estimates of TFP growth in the family sector, and the indirect method used for the US cannot be used because there is nothing comparable to Ruggles's self-employment data for England over this period. We must infer the values of A/D from historical accounts of the relative wages in the cottage industry, a uniquely prominent feature of family production in England during this period.

An increase in the relative TFP of England's family sector is justified by the likelihood that productivity in the cottage industry grew faster than in other sectors during their first industrial revolution. Consider the following quote from William Radcliffe.

> "In the year 1770, …the father of a family would earn from eight shillings to half a guinea at his loom, and his sons, if he had one, two, or three along side of him, six or eight shillings each per week…from the year 1770 to 1788 a complete change had gradually been affected in the spinning of yarns…[O]ur family and some of the others in the neighborhood during the latter half of the time, earned from three to four fold wages [in weaving] to what the same family had heretofore done…The next fifteen years, viz. from 1788 to 1803, which fifteen years I call the golden age of this great trade, which has ever since been in a gradual decline…the price of labour only rose to five times the amount ever before in this sub-division, every family brings home weekly 40, 60, 80, 100, or even 120 shillings per week!!! (Harley, 1998, p. 63)."

Harley regards this fivefold increase in wages to be atypical. His conclusion was that weaving costs in 1815 were at least twice what they were in 1770. However, other historical accounts are in line with Radcliffe's recollection.[22]

Inflation was a little over one percent from 1770 to 1815 (Clark, 2005, Table 3). A doubling of nominal wages would have generated an annual real wage increase of

[22] See the historical accounts from www.cottontimes.co.uk/workers1.html and www.victorianweb.org/history/work/nelson.html.

just under 0.5%. This is very close to the real wage increases of building workers over this period (Clark, 2005, Table 3). However, a four-fold increase in nominal wages suggests a gain in real wages of over 1.5% annually, a full percentage point above those of building workers. Since Harley's conclusion was that wages increased at least twofold, it is quite likely that productivity in the cottage industry rose relative to productivity elsewhere in the economy.

For *Experiment #1*, we assume that relative TFP in family production grew 0.5% higher than in the rest of the economy from 1760 to 1820, remained constant until 1840, and fell after 1840 (assuming no growth in family TFP) rate as formal TFP until 1840, and did not grow at all after 1840. In *Experiment #2*, we maintain a constant value of relative TFP from 1740 to 1820. This was possible if other aspects of family production, such as agriculture, experienced a relative decline in productivity that offset the rise in the relative productivity of the cottage industry.

6.3 Direct private cost of schooling

From (7b), one sees that early schooling is solely driven by the private cost of schooling. Unfortunately, little is known about primary school fees before the 19th century. However, there is evidence that schooling and literacy rates rose from 1760 to 1820 (Crafts, 1997, Table 1). Literacy rates increased by 12% over this 60 year period, rising at a steady pace with no clear acceleration or deceleration.

Since it is well known that there was little government support for education in England until the mid-19th century, the rise in schooling and literacy was likely due to expanding local endowments and increasing support from religious groups. We assume that the cost of schooling fell so as to generate a 4% increase in human capital for each 20 year period from 1760 to 1820. In 1820, the level of schooling is assumed to reach exactly $\bar{s}$, so that $\pi_{1820} = (\theta\tau/(1-\theta))1/\bar{s}$. This is four times the value for π in the US in 1800, consistent with the well known fact that public support for education was much greater in the US than in England until the beginning of the 20th century (Lindert,, 2004).

The initial price of schooling in 1840, the first period of the O-regime, was then set to generate fertility levels equal to the simulated value in 1820. The initial value for π in the O-regime was $\pi_{1840} = 0.75\pi_{1820}$, still about three times the US value for 1800. After 1840, we assumed the same linear decline as was assumed for the US, with the private price of schooling reaching zero eight periods later, at the end of the 20th century.

6.4 Relative productivity of children (γ)

For *Experiment #1* we assume that γ is constant throughout at the U.S. value of 0.28. In *Experiment #2* we consider the possibility that γ rose from 1740 to 1820, before falling back to US levels in 1840, after the cottage industry declined and the employment opportunities for children in factories and mines were curtailed by child labor laws.

From 1740 to 1820 there was a decline in agriculture and an expansion in the cottage industry, as well as the appearance of early factories. This caused a decline in employment opportunities for children in agriculture, but an expansion in both the cottage industry and factories (Cunningham,, 1990, Horrell & Humphries,, 1995, Hudson & King,, 2000). The earnings of children relative to adults were significantly higher in the factories, and especially the cottage industry, than in agriculture (Horrell & Humphries, 1995, Table 5). Thus, as the employment of children shifted from agriculture to domestic and formal industry, the relative productivity of children rose.

We quantify this effect by allowing γ to rise from 1740 to 1820.[23] We begin with a γ of 0.26 in 1740, and then let γ rise linearly to 0.30 by 1820.[24] In 1840, after the cottage industry began to decline, we set γ back to the U.S. value of 0.28 and kept it constant.

6.5 Initial k

We set the initial pre-tax return to capital to be 10% in 1740 and found that there was no downward trend in interest rates until after 1940. Thus, interest rates were about 2 percentage points lower than those simulated for US over the 19th century. This is consistent with the fact that government debt in England was lower than in the US in the 19th century. Outside of war-times, the interest rate on government debt in England during the period was about 2.5% (Barro,, 1993, Chapter 12 compared to about 5% in the US Wallis, 2000, Figure 2). Since simulated interest rates were trendless over the period, physical capital intensity will not play a role in explaining any trends in fertility.

6.6 Qualitative properties of the model

In the Y-regime (1740–1820), a constant growth rate in human capital would leave the relative productivity of children, and the net cost of children, unaffected. So, in *Experiment #1*, where γ is also constant, the fundamental driver is the relative value of family production. A rise in the *level of schooling*, and current earnings, lowers the relative value of family production and causes fertility to decline. However, an increase in the growth rate of TFP (holding constant the relative value of TFP in the informal sectors) will raise fertility because it increases worker productivity in all sectors and the value of future family production relative to current wages. This is reinforced by the rise in the relative TFP in family production over this period, which shifts employment to family production. Thus, for fertility to rise over the Y-regime, the *overall* pace of technological progress must rise and/or the *relative* value of family production must rise because of technological production that was biased toward family production. In *Experiment #2*, the relative TFP in family production is kept constant but γ starts low and then rises. This lowers the net cost of fertility over time, and causes fertility to rise.

In the O-regime (1840 – 1940), the net cost of children will rise because the schooling of parents will rise relative to their working age children. However, the direct cost of schooling young children is falling over this period, which reduces the overall cost of having children. Thus, the net cost of children over this period may rise or fall. The relative value of family production in the numerator should fall for three reasons: a decline in the rate of technological change in the formal sector, a decline in the relative productivity in family production, and a rise in the level of schooling and wages.

23 Horrell and Humphries (1995) argue that the rise in the demand for child labor in industry peaked in the 1820s.

24 We set γ lower than in the US in 1740 because at that time the vast majority of child labor in England was in agriculture and English agriculture lacked the southern crops of the US, where children were relatively more productive (Goldin & Sokoloff, 1984). We set γ higher than in the US in 1800 and 1820 because the data from Horrell and Humphries (1995) show that the relative earnings for children were dramatically higher in the cottage industry, and thus could easily have exceed those in the US on average, where the vast majority of child labor was in agriculture.

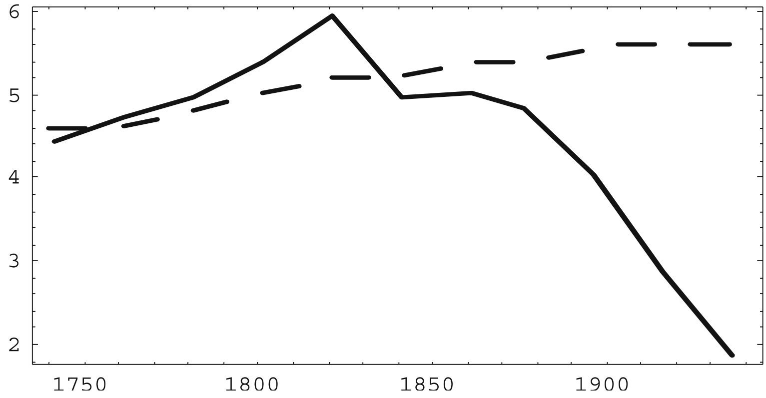

Fig. 5 England fertility 1740 to 1940—no compulsory schooling. *Notes*: Actual fertility—solid line, simulated fertility—dashed line. Actual fertility provided by Woods: see Woods (2000, p. 4, footnote 8) for sources

6.7 Experiment #1 rising relative TFP in the family production

Figure 5 reports the attempt to explain the fertility pattern in England from 1740 to 1940. Simulated fertility (solid line) rises modestly from 2.3 to 2.6 from 1760 to 1820, about 60% of the actual rise (dashed line). However, the model fails to generate a downward trend in fertility after 1820, in fact fertility rises slightly. This is due to the fall in the cost of schooling young children. While a fall in the direct cost of schooling children does increase schooling levels of older children, the effects are too weak to offset the effects of falling schooling prices on the net cost of children and fertility. This issue did not arise in the US simulation because the direct costs of schooling young children was significantly lower in the first period of the simulation, and thus their decline had a relatively weaker effect.

Put another way, the rise in the schooling of older children in the early portion of the O-regime is too weak to pull fertility down. Simulated schooling starts at 0.085 in 1820 and does not even reach 0.09 by 1900. As mentioned, England had reached almost universal basic literacy by 1900, about 40 years after the US England enrollment rates in 1900 were also very similar to those of the US in 1860 (Lindert, Table 5.1). Accordingly, the schooling of older children in England should have risen fast enough to at least hit 0.10 by 1900 (the US value in 1860).

As discussed in Doepke (2004), one explanation for the rapid rise in schooling and literacy in England in the second half of the 19th century could have been the passage of several child labor and compulsory schooling laws (Lindert,, 2004; Mitch, 1992). To examine this possibility we exogenously increased s by equal increments of 0.004 from 1840 to 1880. From this 1880 base, schooling then went slightly over 0.10 in 1900 as a result of the model's dynamics. Figure 6 shows that this was enough to explain some of the fertility decline from 1820 to 1940. This finding is consistent with Doepke's conclusion; compulsory education and child labor laws are needed to explain the decline in England's fertility rates in the second half of the 19th century and early 20th century. It also suggests that such laws might be generally needed to speed up a demographic transition during a period when the cost of schooling is falling.

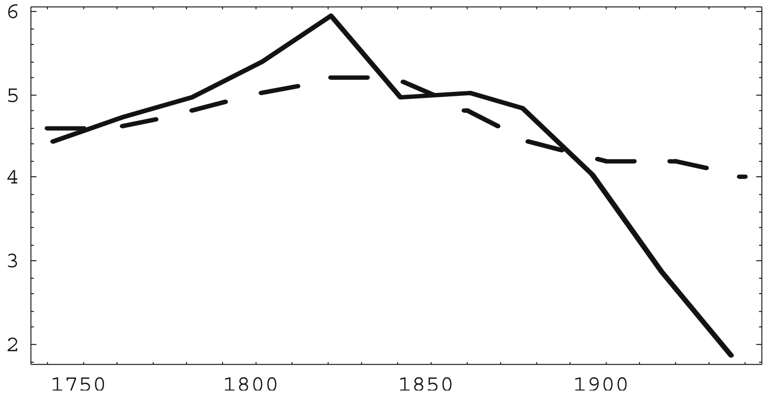

Fig. 6 England fertility 1740–1940—compulsory schooling. *Notes*: Actual fertility—solid line, simulated fertility—dashed line. Actual fertility provided by Woods: see Woods (2000, p. 4, footnote 8) for sources

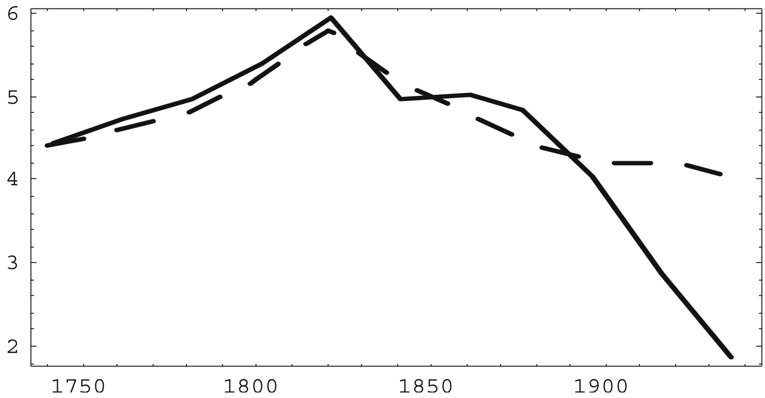

Fig. 7 England fertility 1740 to 1940—changing child productivity. *Notes*: Actual fertility—solid line, simulated fertility—dashed line. Actual fertility provided by Woods: see Woods (2000, p. 4, footnote 8) for sources

6.8 Experiment #2 rising relative productivity of children

Figure 7 displays the simulation for Experiment #2, for the case where we impose compulsory schooling from 1840 to 1880 as in Fig. 6. The rise in γ produces a rise in fertility that is very similar to the rise observed in the data. Once in the O-Regime, since γ is assumed to be constant at the US value, the model's predictions are identical to those in Fig. 6. Thus, some combination of rising relative TFP in the family sector and rising relative productivity, in domestic and formal industries, could reasonably explain the rise in fertility over the period 1740 to 1820. However, compulsory schooling and child labor laws are needed to explain the rapid rise in schooling and the accelerated fertility decline after 1820.

7 Conclusion

In this paper we modeled the role of family-based production and the schooling of older children in a manner consistent with the demographic transition observed in most countries. The model was calibrated to data from the United States over the period 1800–2000. Simulations show that the technologically driven decline in family production caused much of the decline in fertility in the 19th century and the rise in schooling of older children caused approximately the entire decline in the 20th century. The model also generates aggregate labor productivity growth rates that closely match those observed in the United States over the two centuries.

We identify two possible reasons for the rise in fertility in England from 1740 to 1820: (i) a rise in the relative TFP in the family sector (associated with an expansion in the cottage industry) and (ii) a rise in the relative productivity of children. We find that compulsory schooling and child labor laws are needed to explain the accelerated fertility decline after 1820—consistent with Doepke (2004).

Our results are determined, in part, by three features of development not emphasized in the existing literature: (i) a distinction between the schooling of young and old children (ii) a supply-side theory of schooling and its connection with child labor and (iii) the direct effect of family production on fertility.

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Appendix A Proofs of propositions

Proposition 1 Households choose $c_t^y, c_{t+1}^o, f_{t+1}, \bar{s}_t, \tilde{s}_t$, and n_{t+1} to

$$\begin{aligned}
&\max\{\ln c_t^y + \beta \ln c_{t+1}^0 + \psi \ln\left(w_{t+1}h_{t+1}n_{t+1}\right) \\
&+\lambda_t[w_t h_t + n_{t+1} w_t \bar{h}_t \left(T - \bar{s} - \tilde{s}_t\right) + \frac{O_{t+1}^f - \omega_{t+1}}{1+r_{t+1}} - c_t^y - \frac{c_{t+1}^o}{1+r_{t+1}} \\
&-n_{t+1}\left(\tau w_t h_t + p_t\left(\bar{s}_t + \tilde{s}_t\right)\right)] + \bar{\mu}_t\left(\bar{s} - \bar{s}_t\right) + \mu_t^u\left[T - \bar{s} - \tilde{s}_t\right] + \mu_t^l \tilde{s}_t\},
\end{aligned}$$

where $\lambda_t, \bar{\mu}_t, \mu_t^u$, and μ_t^l are non-negative multipliers satisfying complementary slackness conditions with their respective constraints. The first order conditions are

(i) $\frac{1}{c_t^y} = \lambda_t$,

(ii) $\frac{\beta}{c_{t+1}^o} = \frac{\lambda_t}{1+r_{t+1}}$,

(iii) $(1-\rho)A_{t+1}f_{t+1}^{-\rho} = w_{t+1}$,

(iv) $\frac{\psi\theta}{\bar{s}_t} + \lambda_t n_{t+1}\left[\frac{w_t\theta\bar{h}_t}{\bar{s}_t}\left(T - \bar{s} - \tilde{s}_t\right) - p_t\right] - \bar{\mu}_t = 0$,

(v) $\frac{\psi\phi\Phi(\tilde{s}_t+\eta)^{\phi-1}}{1+\Phi(\tilde{s}_t+\eta)^{\phi}} - \lambda_t n_{t+1}\left(w_t\bar{h}_t + p_t\right) + \mu_t^l - \mu_t^u = 0$,

(vi) $\frac{\psi}{n_{t+1}} - \lambda_t \left[\tau w_t h_t + p_t s_t - w_t \bar{h}_t \left(T - \bar{s} - \tilde{s}_t\right)\right] = 0.$

Suppose (A1) holds, $\bar{\mu}_t = 0, \bar{s}_t > 0$, and $\mu_t^l - \mu_t^u > 0, \tilde{s}_t = 0$(the last condition is actually an implication of the first two, as we will verify below). Then one can use (iv) and (vi) to get

(vii) $\bar{s}_t = \frac{\theta\tau}{1-\theta}\frac{w_t h_t}{p_t}$ and

(viii) $\lambda_t n_{t+1} w_t h_t = \psi\left(1-\theta\right) \Big/ \left[\tau - \left(1-\theta\right)\gamma\left(T - \bar{s}\right)\left(\bar{h}_t \Big/ \bar{h}_{t-1}\right)\right].$

To verify $\mu_t^l - \mu_t^u > 0, \tilde{s}_t = 0$, when (vii) and (viii) hold, use $1 + \Phi\eta^{\phi} \equiv 1/\gamma$ and focus on the first expression in (v) to get

$$\frac{\psi\phi\Phi\eta^{\phi-1}}{1+\Phi\eta^{\phi}} = \frac{\psi\phi\left(1-\gamma\right)}{\eta} = \frac{\psi\left(\frac{\theta\tau}{1-\theta}\frac{1}{\bar{s}} + \gamma\right)}{\frac{\tau}{1-\theta} - \gamma\left(T - \bar{s}\right)} \qquad \text{(by (A1))}$$

$$\leq \frac{\psi\left(\frac{\theta\tau}{1-\theta}\frac{1}{\bar{s}_t} + \gamma\frac{\bar{h}_t}{\bar{h}_{t-1}}\right)}{\frac{\tau}{1-\theta} - \gamma\frac{\bar{h}_t}{\bar{h}_{t-1}}\left(T - \bar{s}\right)} \qquad \text{for } \bar{s}_t \leq \bar{s} \text{ and } \bar{h}_t \Big/ \bar{h}_{t-1} \geq 1$$

$$= \frac{\left(1-\theta\right)\psi\left[\frac{p_t}{w_t h_t} + \frac{w_t \bar{h}_t}{w_t h_t}\right]}{\tau - \left(1-\theta\right)\gamma w_t \bar{h}_t (T - \bar{s})/w_t \bar{h}_{t-1}} \qquad \text{(by (vii))}$$

$$= \lambda_t n_{t+1}\left(p_t + w_t \bar{h}_t\right) \qquad \text{(by (viii))},$$

where the strict inequality holds whenever $\bar{s}_t < \bar{s}$ and $\bar{h}_t \Big/ \bar{h}_{t-1} > 1$. This implies that $\frac{\psi\phi\Phi\eta^{\phi-1}}{1+\Phi\eta^{\phi}} < \lambda_t n_{t+1}\left(p_t + w_t \bar{h}_t\right)$, so that $\mu_t^l - \mu_t^u > 0, \tilde{s}_t = 0$ in (v) whenever $\bar{s}_t$ is increasing and less than $\bar{s}$.

Next, note from (vi) that

$$\lambda_t n_{t+1}\left(p_t + w_t \bar{h}_t\right) = \frac{\psi\left(p_t + w_t \bar{h}_t\right)}{\tau w_t h_t + p_t \bar{s}_t - w_t \bar{h}_t\left(T - \bar{s}\right)} = \frac{\psi\left(\frac{p_t}{w_t h_t} + \gamma\frac{\bar{h}_t}{\bar{h}_{t-1}}\right)}{\tau + \frac{p_t \bar{s}_t}{w_t h_t} - \gamma\frac{\bar{h}_t}{\bar{h}_{t-1}}\left(T - \bar{s}\right)}.$$

If (A2) holds, this expression is decreasing in $p_t / w_t h_t$ at $\bar{h}_t = \bar{h}_{t-1} = \bar{s}^{\theta}$. So, as $p_t / w_t h_t$ falls toward $\frac{\theta\tau}{\bar{s}(1-\theta)}$, $\bar{s}_t$ approaches $\bar{s}$. As $p_t / w_t h_t$ falls below $\frac{\theta\tau}{\bar{s}(1-\theta)}$, then $\lambda_t n_{t+1}\left(p_t + w_t \bar{h}_t\right)$ falls below $\frac{\psi\left(\frac{\theta\tau}{1-\theta}\frac{1}{\bar{s}} + \gamma\right)}{\frac{\tau}{1-\theta} - \gamma(T - \bar{s})} = \frac{\psi\phi(1-\gamma)}{\eta} = \frac{\psi\phi\Phi\eta^{\phi-1}}{1+\Phi\eta^{\phi}}$. The inequality, $\frac{\psi\phi\Phi\eta^{\phi-1}}{1+\Phi\eta^{\phi}} > \lambda_t n_{t+1}\left(p_t + w_t \bar{h}_t\right)$, establishes the condition needed for a positive interior solution in (v). Thus, once this condition is met, $\mu_t^l - \mu_t^u = 0, \tilde{s}_t > 0$, until $\tilde{s}_t$ hits the upper bound, $T - \bar{s}$. When the upper bound is hit, then $\mu_t^u > 0, \tilde{s}_t = T - \bar{s}$.

Proposition 2 In the Y-regime, (2b) is (vii) from Proposition 1. Next, use (i) to write (vi) as $\psi c_t^y = n_{t+1}\left[\tau w_t h_t + p_t s_t - w_t \bar{h}_t\left(T - \bar{s} - \tilde{s}_t\right)\right]$ and substitute into the lifetime-budget constraint, along with (ii), to get $c_t^y = \left[\frac{1}{1+\beta+\psi}\right]\left[w_t h_t + \frac{(1-\mu_{t+1})\left(O_{t+1}^f - \omega_{t+1}\right)}{1+r_{t+1}}\right].$

Also note, using (iii), that $(1-\rho)\,A_{t+1} f_{t+1}^{1-\rho} = w_{t+1} f_{t+1} \equiv \omega_{t+1}$. So, $(1-\rho)\,O_{t+1}^f = \omega_{t+1}$

and $O^f_{t+1} - \omega_{t+1} = \rho O^f_{t+1}$. Substituting this expression into the one for c^y_t above, and then substituting the resulting expression into (viii), gives (2a).

In the O-regime, combining (i), (v), and (vi) gives

$$n_{t+1} = \frac{\psi \left(1 - \frac{\phi}{1 + \frac{1}{\Phi(\tilde{s}_t + \eta)^{\phi}}}\right) \left(\frac{c^y_t}{w_t h_t}\right)}{\tau + \frac{p_t(\bar{s} - \eta)}{w_t h_t} - \frac{T + \eta - \bar{s}}{1 + \Phi(\tilde{s}_{t-1} + \eta)^{\phi}}}.$$

Using the same approach to solve for c^y_t as in the Y-regime, and then substituting the resulting solution into the expression for n_{t+1} above, gives (3a).

To get (3b), use the ratio of (v) and (vi) to write

$$\left[1 - \phi + \frac{1}{\Phi\left(\tilde{s}_t + \eta\right)^{\phi}}\right] \tilde{s}_t + \left[1 + \frac{1}{\Phi\left(\tilde{s}_t + \eta\right)^{\phi}}\right] \eta$$
$$= \frac{\phi \left[\tau + \frac{p_t \bar{s}}{w_t h_t} - \frac{T - \bar{s}}{1 + \Phi(\tilde{s}_{t-1} + \eta)^{\phi}}\right]}{\frac{p_t}{w_t h_t} + \frac{1}{1 + \Phi(\tilde{s}_{t-1} + \eta)^{\phi}}},$$

which implicitly defines $\tilde{s}_t$ as a function of $\tilde{s}_{t-1}$ and $p_t / w_t h_t$. One can show that the LHS is strictly increasing in $\tilde{s}_t$. Thus $\tilde{s}_t$ must increase if the RHS increases. An increase in $\tilde{s}_{t-1}$ raises the RHS and $\tilde{s}_t$. (A2) is sufficient to show that the RHS is decreasing in $p_t / w_t h_t$. This implies that $\tilde{s}_t$ is decreasing in $p_t / w_t h_t$, which completes the derivation of (3b).

Finally, evaluating (3a) at $\tilde{s}_t = 0$ and $p_t / w_t h_t = \theta\tau / \bar{s}\,(1 - \theta)$ gives

$$n_{t+1} = \frac{\psi\,(1 - \theta)\,(1 - \phi\,(1 - \gamma))\left(\frac{c^y_t}{w_t h_t}\right)}{\tau - \gamma\,(1 - \theta)\,(T - \bar{s}) - \left(\frac{\theta\tau}{1 - \theta}\frac{1}{\bar{s}} + \gamma\right)\eta}.$$

Using (A1), the denominator can be written as $\left[\tau - \gamma\,(1 - \theta)\,(T - \bar{s})\right](1 - \phi\,(1 - \gamma))$, which shows that (3a) collapses to (2a) at $\tilde{s}_t = 0$ and $p_t / w_t h_t = \theta\tau / \bar{s}\,(1 - \theta)$. Thus, any change in n_{t+1}, as one crosses from one regime to another, is due to rising $\tilde{s}_t$ and falling $p_t / w_t h_t$.

A.1 The difference equation system

(A3), (2b), and Proposition 1 establish that the critical value for π_t that demarcates the two schooling regimes is $\theta\tau / \bar{s}\,(1 - \theta)$. When $\pi_t \geqslant \theta\tau / \bar{s}\,(1 - \theta)$, the economy is in the Y-regime. When $\pi_t < \theta\tau / \bar{s}\,(1 - \theta)$, the economy is in the O-regime. Using (A3)–(A5) the difference equation system can be written as

$$n_{t+1} = \frac{(1 - \theta)\,\psi \left(1 + \frac{(1 + d_{t+1})\left[\frac{A_{t+1}}{D_{t+1}}\right]^{\frac{1}{\alpha}}}{\bar{s}^{\theta}_{t-1}(1 - \alpha)k_t}\right)}{(1 + \beta + \psi)\left(\tau - (1 - \theta)\gamma(T - \bar{s})\left(\bar{s}_t / \bar{s}_{t-1}\right)^{\theta}\right)},$$

$$\bar{s}_t = \frac{\theta\tau}{(1-\theta)}\frac{1}{\pi_t}, \qquad \text{for } \pi_t < \frac{\theta\tau}{1-\theta}\frac{1}{\bar{s}},$$

$$n_{t+1} = \frac{\left(1 - \frac{\phi}{1+\frac{1}{\Phi(\tilde{s}_t+\eta)^{\phi}}}\right)\psi\left(1 + \frac{(1+d_{t+1})\left[\frac{A_{t+1}}{D_{t+1}}\right]^{\frac{1}{\alpha}}}{\gamma\bar{s}^{\theta}(1+\Phi(\tilde{s}_{t-1}+\eta)^{\phi})(1-\alpha)k_t}\right)}{(1+\beta+\psi)\left(\tau + \pi_t(\bar{s}-\eta) - \frac{T+\eta-\bar{s}}{1+\Phi(\tilde{s}_{t-1}+\eta)^{\phi}}\right)},$$

$$\tilde{s}_t = g\left(\tilde{s}_{t-1}, \pi_t\right), \qquad \text{for } \pi_t \geq \frac{\theta\tau}{1-\theta}\frac{1}{\bar{s}}, \qquad \text{and for all } \pi_t,$$

$$\text{with } k_{t+1} = \frac{1}{n_{t+1}l_{t+1}h_{t+1}}\left\{\frac{\beta h_t}{1+\beta+\psi}\frac{(1-\alpha)k_t^{\alpha}}{1+d_{t+1}} - \frac{1+\psi}{1+\beta+\psi}\left[\frac{A_{t+1}}{D_{t+1}}\right]^{\frac{1}{\alpha}}\right\},$$

$$l_{t+1} = 1 + n_{t+2}(T - \bar{s} - \tilde{s}_{t+1})\frac{\bar{h}_{t+1}}{h_{t+1}} - \frac{1}{n_{t+1}h_{t+1}}\left[\frac{A_{t+1}}{D_{t+1}}\right]^{\frac{1}{\alpha}}, \qquad \text{for all } \pi_t.$$

Appendix B: Table 1

Sources for Table 1: Clark (2005)—C, Cipolla (1969)—CP1, Cipolla (1974)—CP2, Crafts (1995)—CR1, Crafts (1997)—CR2, Galor (2005)—G, Greenwood and Seshadri (2002)—GS1, Greenwood and Seshadri (2002)—GS2, Soltow and Stevens (1981)—SS, Tan and Haines (1984)—TN, Woods (2000)—W

England: *Fertility decline*—C (Figure 1) and W (Figure 1.1), *primary school enrollment rates*—CR1 (Table 2—average of entries for 1801–1831 and 1831–1873), *economic transformation*— CR1 (Table 2) and G (Figure 2.14), and *literacy rate*— CR2 (Table 1).

Germany: *Fertility decline and school enrollment rates*—TN (Table A.2, p. 79), *economic transformation*—CP2 (Figure 4) and G (Figure 2.14), and *literacy rate*—CP1 (Table 2.4).

USA: *Fertility decline and economic transformation*—GS1(Figure 1), GS2(Figure 2), and *literacy rates*—SS (Tables 2.1–2.2).

Japan: *Fertility decline and school enrollment rates*—TN (Table A.5) and *economic transformation*—CP2 (Figure 4).

Mexico: *Fertility decline and school enrollment rates*—TN (Table B.6, p. 88), and *economic transformation*—G (Figure 2.14).

India: *Fertility decline and school enrollment rates*—TN (Table B.8, p.89), and *economic transformation*—CP2 (figure 4) and G (Figure 2.14).

Third World: *Fertility decline and school enrollment rates*—TN (Table 6, p. 50), and *economic transformation*—G (Figure 2.14).

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